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On the tension test as a means of characterizing fibre composite failure mode

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A model of composite tensile failure, which has been utilized previously in analysing modes of composite fracture [1], is extended to describe fracture in systems not considered previously. This model, based on tensile testing with a stiff, elongation rate controlled machine, predicts a strain concentration in the vicinity of a fibre break during such testing. The magnitude of the stress increase associated with this strain concentration is analytically predicted and compared with experimental data. The existence of this strain concentration can lead to phenomenologically different modes of composite failure. The type of failure observed depends upon the properties of the composite constituents as well as the composite parameters, fibre volume fraction (V_f), number of fibres in the composite (N) and ratio of specimen length to fibre ineffective length (L/δ).

1. Introduction

The tension test is utilized frequently not only to determine the flow properties of fibre composites but to characterize composite failure mode as well. In a previous study [1], we had noted some difficulties associated with characterizing intrinsic fracture behaviour of composites by their observed mode of fracture in a tensile test. We were especially concerned with the three types of fracture mode illustrated schematically in Fig. 1. These modes we have called statistical (curve A, Fig. 1), stepwise (curve C) and instantaneous (curve B). In this previous work a model was developed which was applicable to tension testing with a stiff machine under controlled elongation rate conditions, which showed that the three types of failure mode could be observed in material in which there was no fundamental difference in fracture mode. The physical basis of this model is that tensile failure of an individual fibre in a composite is associated with a drop in tensile load under such testing conditions. Furthermore, the condition of instantaneously constant specimen length requires that the longitudinal contraction due to the load drop in the region of the specimen removed from the fibre break be accompanied by an extension in the region of the specimen in the vicinity of the fibre break. This causes an effective strain concentration in this region which, depending

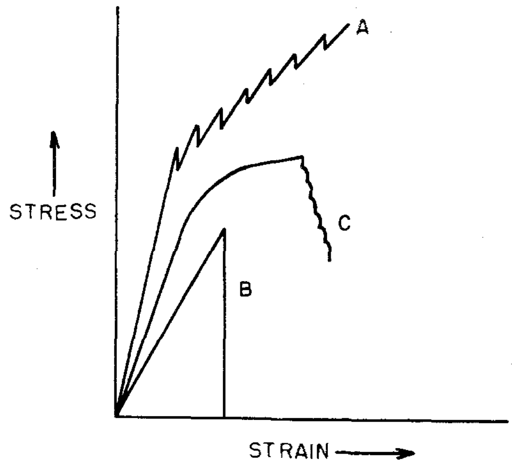

Figure 1 Schematic representation of tensile failure modes frequently observed in tensile failure of fibre composite materials. A, statistical failure, B, instantaneous failure, C, stepwise failure.

on the distribution in fibre strains-to-failure (or tensile strengths) in the composite, may be sufficient to break one or more additional fibres in this region. If this distribution is narrow, the strain concentration can lead to instantaneous failure. However, if it is broad, additional composite elongation is required for composite failure and this leads to either a stepwise or statistical fracture mode. Thus several different

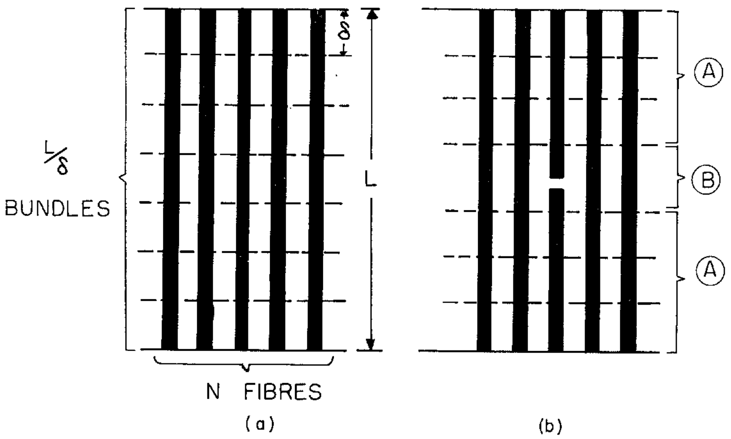

Figure 2 (a) Model of composite tested in uniaxial tension. The composite of length L is divided into L/δ bundles of length δ, the ineffective length. (b) Composite after failure of one link in region B illustrating extension of composite in this region and elastic contraction in region A as a result of load drop upon failure of the first link.

types of phenomenological fracture behaviour may be observed even though there is no fundamental difference in fracture characteristics. Furthermore, the model emphasizes that instantaneous failure need not be associated with dynamic effects.

This model was utilized to characterize the fracture behaviour of some eighty-five metal matrix (copper or aluminium)–metal filament (molybdenum or tungsten) composites tensile tested over a range of temperature from 77 to 298 K. Variations in fibre properties were determined and fracture mode was predicted to be either of the stepwise or instantaneous mode depending upon this variation. The observed fracture modes were found to be in agreement with those predicted in over 80% of the samples tested.

Since the previous work was restricted to systems characterized by relatively narrow distributions in fibre mechanical properties, only the instantaneous and stepwise failure modes were observed. The objective of this paper is to further verify the model and to extend it to systems in which the fibres are characterized by broader distributions in mechanical properties. Additionally, some comments are made comparing composite fracture modes when tested under controlled load as opposed to controlled elongation conditions.

2. Model of composite failure

The model which has been utilized to describe composite failure is based on the schematic representation of Fig. 2 which shows a composite specimen of length L containing N continuous uniaxially aligned fibres. As the specimen is loaded in uniaxial tension, the stress and strain in the fibres increase until the weakest or least ductile fibre fails. As shown in Fig. 2, once this fibre breaks it is ineffective in carrying its share of the load only over a region of distance $\pm\ \delta/2$ from the fibre break, where δ is the fibre ineffective length. In the simplified model illustrated in Fig. 2, the contribution of the matrix to the strength of the composite is not considered and the force balance illustrated shows that the fibres in region B are supporting a stress higher than those in region A by the factor $N/(N-1)$.

Under the experimental conditions applying in most tensile tests, i.e. a stiff elongation rate controlled machine, the failure of the first fibre results in a drop in applied load. This load drop and corresponding decrease in stress result in an elastic contraction in region A which, as mentioned previously, is accompanied by an extension in region B and a corresponding strain concentration there.

If the contribution of the matrix to composite strength is neglected, one would intuitively expect the load to drop to a level $(N - 1/N)$ times that load carried by the composite prior to the first fibre break where N is the number of fibres in the composite. Further, it is expected that the fibres in region B containing the broken fibre would be carrying the same stress as they did prior to the first break. Indeed such a situation would exist if the composite behaved as a bundle, i.e. if the ratio L/δ approaches unity. Because of the strain concentration in region B, however, the fibres there are not only carrying a higher stress than those in region A, but they are also stressed to a higher level than that sustained prior to the first fibre failure.

A force balance taken at the boundary

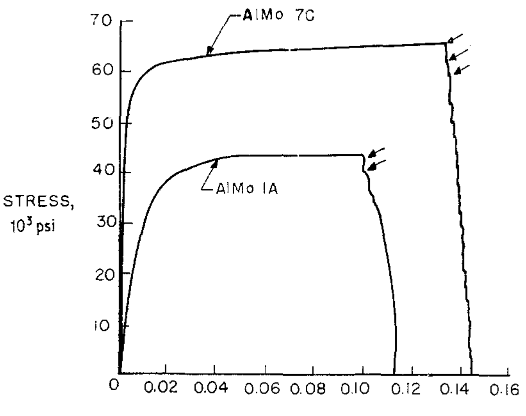

Figure 3 Several stress-strain curves of Al–Mo fibre composites which failed in a step-wise manner. Arrows indicate individual fibre breaks.

between regions A and B together with the condition that the contraction in region A must be compensated by an extension in region B (instantaneously constant specimen length) allows the derivation of an expression for the incremental strain, $\Delta\epsilon_B$, in region B associated with the first fibre break [1],

$$\Delta\epsilon_B = \frac{V_f\sigma_{f1}(L/\delta - 1)}{N[E_c + K_c(L/\delta - 1)] - V_fK_f(L/\delta - 1)}, \quad (1)$$

where V_f = fibre volume fraction, σ_{f1} = fibre stress prior to first fibre break, E_c = composite elastic modulus, K_c, K_f = slope of composite, fibre stress-strain curve ($d\sigma/d\epsilon$) at the strain of the first fibre break.

When considering composites containing ductile fibres, this strain concentration is used directly as a comparison to the spread in strains-to-failure of individual fibres. For composites containing brittle fibres, a more useful parameter is the stress concentration arising from the incremental strain, $\Delta\epsilon_B$ [1];

$$\frac{\sigma_{fB}}{\sigma_{f1}} = \frac{N[E_c + K_c(L/\delta - 1)]}{N[E_c + K_c(L/\delta - 1)] - V_fK_f(L/\delta - 1)}, \quad (2)$$

where σ_{fB} is the fibre stress in region B subsequent to the first fibre break. It is important to note that the strain concentration (Equation 1) is greater than zero, and the stress concentration (Equation 2) is greater than unity for all cases except the limiting ones of $N \to \infty$ or $L/\delta \to 1$. Additionally, the stress and strain concentrations depend upon fibre volume fraction both explicitly and also implicitly through the parameters E_c and K_c and the concentrations increase monotonically with increasing V_f. As mentioned previously, that this model is qualitatively correct has been demonstrated by its success in predicting fracture mode. It can be subject to a more stringent test, however, by accurate measurement of composite stress prior and subsequent to a fibre break for composites which fail in a step wise manner. Several such stress-strain curves are shown in Fig. 3 and, by use of suitable load scales, the ratio of composite stress after and before a fibre break can be measured to about one or two parts in a thousand. If there were no strain concentration in region B, the ratio of the composite stress subsequent to the first fibre failure (σ_{cA}) compared to that prior to the first fibre failure (σ_{c1}) is readily calculated. The force prior to the first fibre failure, F_1, is given as

$$F_1 = Na_f\sigma_{f1} + A_m\sigma_m = \sigma_{c1}A_c, \quad (3)$$

where A_c is composite cross-sectional area, A_m the matrix cross-sectional area, and a_f the area of an individual fibre. If no stress concentration occurs due to a break in region B, the load carried by the fibres drops by the factor $1/N$ i.e.

$$F_B = (N - 1)a_f\sigma_{f1} + A_m\sigma_m = \sigma_{cA}A_c. \quad (4)$$

The term $\sigma_{cA}A_c$ arises from the appropriate force balance across regions A and B. On rearranging, Equations 3 and 4 reduce to

$$\frac{\sigma_{cA}}{\sigma_{c1}} = 1 - \left(\frac{V_f}{N}\right)\frac{\sigma_{f1}}{\sigma_{c1}}. \quad (5)$$

For the case where the strain concentration is expressed correctly by Equation 1, the ratio σ_{cA}/σ_{c1} is given by Equation 6 (see below).

Comparison of observed experimental values of σ_{cA}/σ_{c1} to those calculated using Equations 5 and 6 are given in Table I for a number of composites that were observed to fail in a step-wise manner. It is seen that, except in a few cases, σ_{cA}/σ_{c1} is more closely given by Equation 6 than

$$\frac{\sigma_{cA}}{\sigma_{c1}} = \frac{N[E_c + K_c(L/\delta - 1)] - V_f\left[E_c\left(\frac{\sigma_{f1}}{\sigma_{c1}}\right) + K_f(L/\delta - 1)\right]}{N[E_c + K_c(L/\delta - 1)] - V_fK_f(L/\delta - 1)}. \quad (6)$$

TABLE I

Specimen	Fibre	Matrix	N (number of fibres in composite)	V_f	σ_{cA}/σ_{c1} (Eq. 5)	σ_{cA}/σ_{c1} (Eq. 6)	σ_{cA}/σ_{c1} (observed)
2D	Mo	Al	19	0.20	0.962	0.966	0.914
1A	Mo	Al	19	0.24	0.962	0.975	0.970
1C	Mo	Al	19	0.24	0.970	*	0.932
34B	Mo	Al	19	0.34	0.955	0.993	0.960
35B	Mo	Al	18	0.34	0.954	0.959	0.957
19B	Mo	Al	19	0.38	0.957	0.962	0.965
7B	Mo	Al	19	0.43	0.960	*	0.964
7C	Mo	Al	19	0.43	0.954	0.958	0.963
8B	Mo	Al	19	0.44	0.960	*	0.964
22A	W	Al	19	0.40	0.950	0.954	0.952
22B	W	Al	19	0.40	0.950	0.961	0.960
22C	W	Al	19	0.40	0.950	0.972	0.967
21B	W	Al	19	0.37	0.954	*	0.961
3A	W	Cu	19	0.26	0.952	0.970	0.942
27A	Mo	Al	37	0.40	0.976	*	0.982
27B	Mo	Al	37	0.40	0.976	*	0.980
27C	Mo	Al	37	0.40	0.976	*	0.978
27D	Mo	Al	37	0.40	0.976	*	0.983

*Composite tensile tested with non-heat-treated Al matrix. At the small values of composite strain-to-failure observed in these specimens, K_f and K_m could not be measured accurately enough to predict σ_{cA}/σ_{c1} utilizing Equation 6.

Equation 5 but most importantly the observed value of σ_{cA}/σ_{c1} is almost always greater than that predicted by Equation 5 indicating that a strain concentration of the type suggested does exist.

These results further substantiate the previous arguments that the stress and strain concentrations produced by a fibre break in a tensile test on a composite can measurably influence fracture mode. Indeed, depending upon the mechanical characteristics of the composite constituents, the fibre volume fraction, the number of fibres in the composite and the spread in strains-to-failure and tensile strengths of the fibre, the type of fracture can vary from statistical to instantaneous. The higher the fibre volume fraction, the fewer the number of fibres in the composite, the greater the homogeneity in fibre properties and the smaller the fibre ineffective length, the greater is the tendency for instantaneous failure. Since the previous work considered only the stepwise and instantaneous failure modes, the Appendix gives a more detailed analysis of the transition from statistical to stepwise to instantaneous failure as a function of the above mentioned parameters for the case of brittle or high work hardening fibres embedded in a matrix which contributes little to composite strength.

One final comment is in order. Most tensile testing of materials is conducted on an elongation controlled machine. In practice, however, load control is often the service condition. The model described here can be altered for this situation by assuming load, rather than specimen length, is instantaneously fixed upon failure of the first fibre. Thus the fibre stress in region A (Fig. 2) remains the same before and after the break whereas the fibre stress in region B is increased. Under this condition, the incremental increase in strain and the corresponding stress concentration are found to be

$$\Delta\epsilon_B = \frac{V_f\sigma_{f1}}{NK_c - V_fK_f}\,, \tag{7}$$

and

$$\frac{\sigma_{fB}}{\sigma_{f1}} = \frac{NK_c}{NK_c - V_fK_f}\,. \tag{8}$$

For brittle fibres ($K_f = E_f$; $K_c \simeq V_fE_f$), Equations 1 and 7, and 2 and 8 give approximately the same result for values of $L/\delta \gtrsim 6$. For the case of ductile fibres exhibiting little or no work hardening capability ($K_f \ll E_f$), the incremental increase in strain in the load controlled case is so great that the instantaneous mode will always be preferred over the stepwise mode, i.e. as the parametric variables favouring instantaneous failure are increased, a transition from the statistical failure mode to the instantaneous failure mode will occur directly.

3. Conclusions

The use of a tension test to characterize fibre composite failure modes may be ambiguous. This is because under constant elongation rate testing conditions in a stiff machine, an effective strain concentration exists in the vicinity of the first fibre break. The magnitude of this concentration can be estimated analytically and is shown to be in reasonable agreement with measured values of the increase in stress resulting from the strain concentration.

Because of the existence of this concentration, a variety of phenomenologically different modes of composite failure may be observed in tensile testing even when there is no fundamental difference in fracture behaviour and, therefore, the results of many experimental studies utilizing tensile testing cannot be directly related to service conditions in which load, rather than elongation, control is operative.

Appendix: Instantaneous-stepwise-statistical transition in failure mode

This Appendix is an epexegesis of the discussion in the text which indicated the possibility of statistical failure in specimens characterized by narrow distributions in fibre strengths. This analysis will consider only the elongation controlled case although the reasoning is equally applicable to the load controlled situation. Since the statistical mode should occur only for composites containing brittle or high work-hardening fibres [1], this section will be based on the assumption of a specimen containing only brittle fibres ($K_f = E_f$). Additionally, the simplifying assumption will be made that the matrix contribution to the composite modulus may be neglected, i.e. $E_c \simeq V_f E_f$. With these assumptions Equation 2 reduces to

$$\frac{\sigma_{fB}}{\sigma_{f_1}} = \frac{(N)L/\delta}{[(N-1)L/\delta]+1}\,. \qquad \text{(A1)}$$

An expression for the stress σ_{fA} in region A after the first fibre break is similarly found to be (cf. Fig. 2)

$$\frac{\sigma_{fA}}{\sigma_{f_1}} = \frac{(N-1)L/\delta}{[(N-1)L/\delta]+1}\,. \qquad \text{(A2)}$$

Equations A1 and A2 may be generalized to determine the stresses after a total of Y breaks in region B with no breaks in region A. These expressions are found to be,

$$\frac{\sigma_{fBY}}{\sigma_{f_1}} = \frac{N(L/\delta)}{[(N-Y)L/\delta]+Y}\,. \qquad \text{(A3)}$$

and

$$\frac{\sigma_{fAY}}{\sigma_{f_1}} = \frac{(N-Y)L/\delta}{[(N-Y)L/\delta]+Y}\,. \qquad \text{(A4)}$$

where σ_{fAY}, σ_{fBY} = fibre stress in region A, B after Yth break in region B.

As discussed in the development of the model, the stress concentration in region B arising from the first break may be sufficient to break additional links in that region. The new stress in region B resulting from these additional breaks may then be calculated (Equation A3) and the process iterated. (The logic of this and the following steps is shown schematically in Fig. 4.) If subsequent stresses are always sufficient to break additional links in region B, instantaneous failure results*. If the initial stress concentration is not sufficient to break additional links in region B, the applied load (elongation) must be increased. The remaining fibre failures may, however, still be restricted to bundle B. This occurs when the load required to break an additional link in region B is less than that required to break a link in region A. Under such circumstances, a stepwise stress-strain curve results. In most cases, however, after a certain number of links have failed in a stepwise manner, the remaining ones may fail instantaneously.

Conversely, if the load required to break an additional link in region B is greater than that required to break the first link in region A, a statistical failure mode results. This type failure mode is favoured by a wide distribution in link strengths whereas a narrow distribution in strengths will favour instantaneous or stepwise failure.

Characterization of strength distribution

The description of the distribution in strengths of a group of fibres, or a group of fibre links as in the case of the present model, is dependent upon the length of the fibres or links under consideration. (This results from the increasing probability of encountering a flaw with increasing

*It is noteworthy that in all of the cases examined in this analysis, if the stress concentration due to the first fibre break is sufficient to break one or more additional links in region B, instantaneous failure always results.

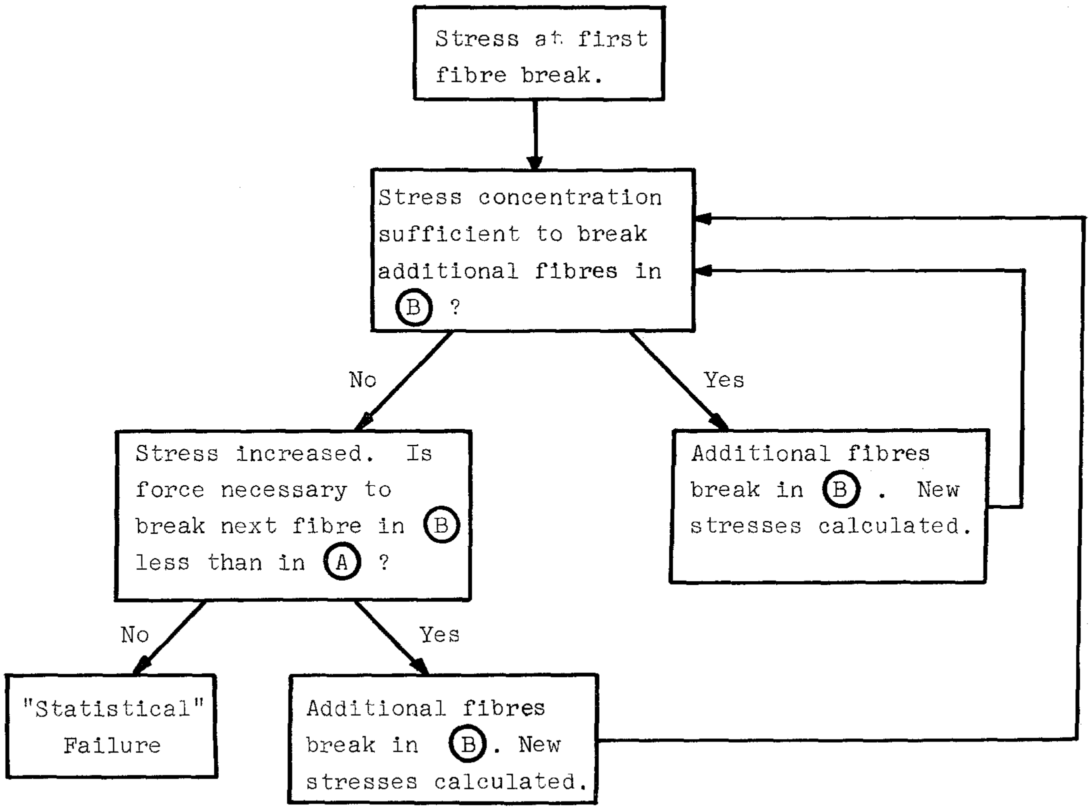

Figure 4 Logic flow chart for analysis of composite tensile failure mode.

length.) An analysis based on a fixed distribution function in link strengths is not a good statistical description because it implies that changing the fibre ineffective length changes the distribution in fibre strengths. In fact, the fibre strength distribution is fixed and changing δ changes the link strength distribution. Since the present analysis is based on the link strength distribution, an expression relating this distribution to that of the fibre strengths is more physically realistic.

The cumulative link strength distribution function $F(\sigma)$ is defined as the fraction of links which fail at a stress less than or equal to σ. The relationship between the fibre cumulative distribution function, $G(\sigma)$, for fibres of length l and the link cumulative distribution function, $F(\sigma)$, for links of length δ is given by [2, 3]:

$$F(\sigma) = 1 - [1 - G(\sigma)]^{\delta/l}. \quad (A5)$$

As mentioned above, for a given $G(\sigma)$, $F(\sigma)$ depends on the value of the ineffective length. Most fibre strength distribution functions are normalized to fibre elements of length equal to their diameter. This fundamental cumulative distribution function can be related to that of the link by realizing that $G(\sigma)$ is given both by

$$G(\sigma) = 1 - [1 - F(\sigma)]^{l/\delta}, \quad (A6)$$

and by

$$G(\sigma) = 1 - [1 - F'(\sigma)]^{l/d_f}, \quad (A7)$$

where $F'(\sigma)$ is the cumulative distribution function for elements of fibre of length equal to their diameter, d_f.

For the present purposes, the approximation of a Weibull distribution [4] was chosen to represent the element distribution function, i.e.

$$F'(\sigma) = (\sigma/\sigma_{max})^m, \quad (A8)$$

where σ_{max} is the theoretical strength and m describes the width of the distribution – large values of m represent a narrow strength distribution. Equations A6 to A8 yield

$$F(\sigma) = 1 - \left[1 - \left(\frac{\sigma}{\sigma_{max}}\right)^m\right]^{\delta/d_f}. \quad (A9)$$

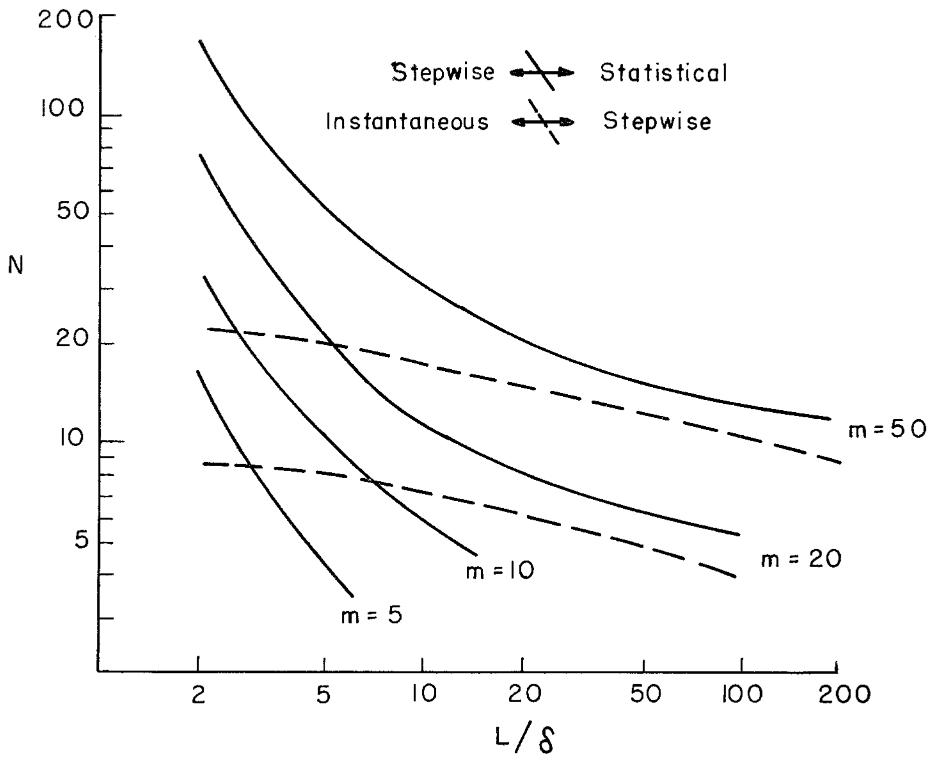

Figure 5 Composite failure modes for specimens containing brittle fibres characterized by a Weibull distribution in element strengths.

The variation of $F(\sigma)$ with δ is noted implicitly in Equation A9.

Determination of failure mode

The stress σ_{f_1} at which the first link is broken may be found by equating the cumulative distribution (Equation A9) to $1/N(L/\delta)$,

$$F(\sigma_{f_1}) = 1 - [1 - (\sigma_{f_1}/\sigma_{max})^m]^{\delta/d} = \frac{1}{(N)L/\delta} \tag{A10}$$

with $N(L/\delta)$ being the total number of links in the model (see Fig. 2). The stress σ_{fB} in region B after the first break and the corresponding cumulative distribution $F(\sigma_{fB})$ were then calculated using Equations A1 and A9. The criterion for failure of an additional link in region B due to the stress concentration is

$$F(\sigma_{fB}) \geq \frac{1}{(N)L/\delta} + \frac{1}{N-1}\left[1 - \frac{1}{(N)L/\delta}\right] . \tag{A11}$$

In this expression $1/(N)L/\delta$ is the probability of the first link failure. The probability of failure of one additional link out of $N - 1$ links remaining in region B is $1/(N - 1)$, but this term must be normalized by the factor in brackets since these $N - 1$ links have a narrower strength distribution than the original $(N)L/\delta$ links. If $F(\sigma_{fB})$ is sufficiently large so that at least one additional link is broken (i.e. Equation A11 is true), the total number of additional links, X, broken by the stress σ_{fB} is found by equating $F(\sigma_{fB})$ to the cumulative distribution function representing $(X + 1)$ breaks in region B, i.e.

$$F(\sigma_{fB}) = \frac{1}{(N)L/\delta} + \frac{X}{N-1}\left[1 - \frac{1}{(N)L/\delta}\right] . \tag{A12}$$

If, however, $F(\sigma_{fB})$ is less than the expression in Equation A11, the applied load (elongation) must be increased to break the next link. In this case, the stress σ_{fB}' necessary to break the next link in region B is determined by equating $F(\sigma_{fB}')$ to the right hand side of Equation A11. If the specimen elongation is increased, however, the stress in region A, as well as that in region B, rises. The possibility exists, therefore, for failure of a link in region A prior to failure of an additional link in region B. Using the same logic which led to Equation A11 the criterion for failure of one of the $N[(L/\delta) - 1]$ links in region A is

$$F(\sigma_{fB}') = \frac{1}{(N)L/\delta} + \frac{1}{N[(L/\delta) - 1]}\left[1 - \frac{1}{(N)L/\delta}\right] . \quad (A13)$$

The applied loads P_A' and P_B' corresponding to σ_{fA}' and σ_{fB}' were determined by multiplying the fibre stress in each region by the number of load carrying members in that region (ignoring the matrix contribution):

$$P_A' = N\sigma_A' A_f , \quad (A14)$$

$$P_B' = (N - 1)\sigma_{fB}' A_f . \quad (A15)$$

Failure of the next link will occur in region B (stepwise failure) if $P_B' < P_A'$. If P_A' is smaller, the next link will fail in region A and a statistical failure mode, in the sense that fibre breaks will not be confined to one cross-section, results.

Using this Weibull link distribution (Equation A9) composite failure modes have been predicted for differing values of the parameters m, N, and L/δ. The results are shown in Fig. 5. For values of $m \leq 2$ (not shown), which would signify a wide distribution in link strengths, statistical failure mode occurs at all values of N and L/δ. As m increases to 5, the distribution of link strengths becomes more narrow, and stepwise failure occurs at small values of N and L/δ. On further increases of m, the boundary line between stepwise and statistical failure modes moves to progressively higher values of N and L/δ. It should be noted that $m = 50$ characterizes a very narrow distribution in link strengths. In all cases, instantaneous failure is observed only for high values of m and for small numbers of fibres, i.e. N less than about 20. It is interesting to note that the use of a Weibull distribution produces no change in the failure mode due to varying the parameter L/δ.

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